Designing Ceramic Components for Structural Applications

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Despite significant advances in the properties of structural ceramic materials in recent years, ceramics are not being utilized to their fullest potential due to their inherent brittle nature and a general lack of understanding of how to design reliable ceramic components. The characteristics of ceramics are much different from those of metals, and the design process must account for those differences. Weibull statistical analysis, in conjunction with finite element analysis, provides a basis for designing reliable ceramic components.

Keywords
ceramic, design, Weibull analysis

What is a Ceramic?

CERAMICS are generally defined as inorganic, nonmetallic materials which are (1) cast from a molten state, or (2) formed and simultaneously or subsequently consolidated by heating. Most ceramics that could be considered for structural applications consist of metals or metalloids combined with oxygen, carbon, nitrogen, or boron. Some examples are aluminum oxide (alumina), zirconium dioxide (zirconia), silicon carbide, and silicon nitride.

Physical Characteristics in General

Ceramics are crystalline in nature as are metals. However, the atoms in ceramics are held together with covalent and/or ionic bonds as opposed to the metallic bonds of metals. The atomic/molecular bonding results in physical characteristics different from those of metallic materials.

In general, ceramics exhibit lower thermal expansion coefficients, lower thermal conductivity, and little or no electrical conductivity compared to metals. These properties vary greatly within the realm of ceramic materials, and are strongly dependent on the type of bonding that is predominant in a material (ionic vs. covalent). For example, covalently bonded ceramics tend to have higher elastic moduli and lower thermal expansion coefficients than ionically bonded ceramics. This is due to the higher interatomic bond strength in covalently bonded structures.

Mechanical Properties in General

Mechanical properties are those properties measured by application of force, such as strength, hardness, toughness, ductility, fatigue, creep, etc. Some mechanical properties make ceramics attractive to the design engineer. Other mechanical properties have prevented ceramics from becoming universally accepted. The mechanical properties of interest for structural applications are discussed below. Comparisons are made to metallic materials for reference.

Elastic Modulus: Ceramics tend to have higher elastic moduli than metals, although the elastic properties vary greatly among the various families of ceramics. Again, the elastic properties are a strong function of the type of bonding that is prevalent in the material. In general, covalently bonded ceramics have higher elastic moduli than ionically bonded ceramics.

Hardness: Ceramics are typically much harder than metals. On the Mohs hardness scale, which ranks materials from 1 to 10 based on their ability to scratch one another, structural ceramics are usually in the 8 to 10 range. Metals rank from 3 to 7 or 8. However, hardness is a very complex property, and is very strongly dependent on the hardness testing method utilized. Hardness tests measure the resistance to penetration (Rockwell, Brinell, Vickers, Knoop), the dynamic or rebound hardness (Shore), or the width of a scratch made with a diamond indentor. Obviously, the properties being evaluated vary with the type of test. In addition, the results vary greatly depending on the type of material being tested.

For example, when a metal is indented with a Vickers (diamond pyramid) indentor, the material first deforms elastically, and then plastically. After the load is removed, there is a certain amount of elastic recovery. Since the hardness is calculated based on the diagonal measurement of the permanent indentation, the measured hardness is a function of the elastic modulus, the yield strength, and the strain hardening rate. When a ceramic is indented with the same Vickers indentor and load, the material initially deforms elastically, after which it cracks. Subsequent indentor penetration serves to propagate the cracks away from the indentation. When the load is removed, there is some elastic recovery. Therefore, in a ceramic material, the measured hardness is a function of the elastic modulus, the fracture toughness, and the crack propagation energy.

It should be obvious that comparison of metallic and ceramic hardness values is inaccurate. In general, ceramics are much harder and resistant to erosion, abrasion, and other types of wear, even though some metals measure "harder" than some ceramics. In general, covalently bonded ceramics are harder than ionically bonded ceramics, due to their higher elastic moduli.

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Strength: There are many different properties relating to material strength. Typically, metals are evaluated for yield strength (a rough estimate of the engineering stress above which stress is not proportional to strain) and ultimate tensile strength (the maximum engineering stress developed before fracture). These properties are obtained for metals by loading a specimen in tension, plotting the stress (load divided by initial area) as a function of strain (change in length divided by initial length), and graphically estimating the yield and ultimate loads, and calculating the stresses.

Because ceramics do not deform plastically, the concept of yield strength is inapplicable. It is possible to determine the ultimate tensile strength of ceramic materials. However, ceramics are very defect-sensitive. The measured ultimate tensile strengths of ceramic materials are much lower than their theoretical strengths due to the inherent presence of defects (porosity, inclusions, etc.). In addition, standard tensile testing equipment is generally incapable of loading a specimen in pure tension, i.e., without developing slight bending stresses. These bending stresses are often called "parasitic" stresses, and result in invalid tensile strength measurements. For these reasons, tensile testing of ceramics is not conducted on a routine basis.

Modulus of Rupture: The strength of ceramics is generally evaluated using a flexural test. The strength value resulting from this test is most commonly called the modulus of rupture (MOR), which is also sometimes referred to as the flexural strength, bending strength, or rupture modulus. The test involves bending a specimen of rectangular cross-section until fracture. Bending is accomplished with either 3- or 4-point loading (see Fig. 1), with the 4-point method being the preferred technique. The specimen is suspended across two round bars, and the load is applied using either one or two round bars positioned on top of the specimen and between the round support bars. The load to cause fracture is recorded, and the modulus of rupture is calculated as the maximum tensile stress in the specimen at fracture per the following equation:

$$MOR = \frac{3[L_1 - L_2]P}{2h^2b} \qquad [1]$$

where

L_1 = distance between support points

L_2 = distance between load points (= 0 for 3-point bend)

P = load

h = height of specimen

b = width of specimen

Since the rupture test technique produces a nonuniform stress distribution with the maximum tensile stress in the outer skin of the specimen, there is less chance that defects in the material will cause "premature" failure, unless they happen to be located in the region of highest stress. Thus, the results of these tests are more consistent than tensile tests. However, the strength limit of the material is not necessarily revealed, because the fracture may have initiated at a defect that was outside the region of maximum tensile stress. This concept is discussed in more detail later.

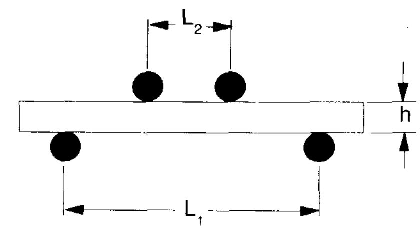

Fig. 1 MOR test dimensions.

Problems can be encountered when comparing rupture test data: 3-point bending concentrates the maximum tensile stress into a very small region in the specimen; 4-point bending distributes stress over a larger volume in the specimen. The effective volumes under tensile stress for 3-point and 4-point bending are calculated using the following equations:[1]

3-point bending:

$$V_e = \frac{V}{2(m+1)^2} \qquad [2]$$

4-point bending:

$$V_e = \frac{V(m+2)}{4(m+1)^2} \qquad [3]$$

where V is L_1hb and m is the Weibull modulus.

The Weibull modulus will be discussed in detail later in the sections dealing with Weibull theory and its application in the design of ceramic parts.

Assuming the Weibull modulus for a given material is 10, the fractional effective volume under tensile stress (V_e/V) for 3-point bending is 0.00413, and for 4-point bending is 0.02479. Therefore, the 4-point bending technique places 6 times as much material under effective tensile stress as the 3-point bending technique. For a material with a Weibull modulus of 30, the 4-point bending technique places 248 times as much material under effective tensile stress.

Because there is a lower probability that a significant defect exists in the smaller stressed region in the 3-point test, the mean strength values obtained are higher than those obtained with the 4-point bending method. At this time, there is no standardization of samples and/or testing techniques used for determining the strength of ceramic materials. The results from tests with different parameters are not equivalent, and strength values from various sources should be compared with extreme caution.

Compressive Strength: Whereas the strength in tension is strongly affected by defects in the material, the compressive strength is much less influenced by defects. Ceramics generally demonstrate compressive strengths many times greater than their rupture moduli. Parts which are loaded only in compression are ideal candidates for ceramic materials.

Toughness: In metals, toughness is measured using several techniques, the most common of which is the Charpy impact test. The Charpy test involves striking a notched rectangular specimen with a weighted hammer and measuring the energy absorbed during fracture. The higher the energy, the tougher the material. The tests are sometimes conducted at numerous temperatures to determine the ductile-to-brittle transition temperature. More recently, fracture toughness properties defined under the field of fracture mechanics (such as K_{IC}, the plane-strain critical fracture stress) have been utilized to evaluate the toughness of materials and their tolerance of defects.

Due to the nature of ionic and covalent bonding, ceramics are inherently brittle. Even unnotched Charpy specimens would demonstrate negligible energy absorption during fracture. Toughness in ceramics is generally reported as a K_{IC} value. However, as in flexural testing, there are many techniques for evaluating fracture toughness, and results reported by different suppliers may not be equivalent. In general, covalently bonded ceramics are more brittle than ionically bonded ceramics.

Ductility: Ductility is the amount of permanent (nonelastic) deformation that a material displays before fracture, and is usually expressed as percent elongation. Most conventional metals display from several percent to as high as 60% elongation. Although ductility is a very important parameter to industries involved in metal forming, in most applications it is used only as an index of the amount of "forgiveness" in a material. A material with a great deal of ductility will "warn" the user that it is being overloaded by deforming measurably before fracturing. A material with little or no ductility breaks with no perceptible warning.

Ceramics in general display no measurable ductility. This is one of the reasons that they have been used sparingly for many applications where they might otherwise provide improved performance over metallic materials.

Weibull Statistics

As was discussed briefly in the section on mechanical properties, the degree to which ceramic materials can withstand loading without fracture depends on the presence of flaws such as inclusions, porosity, and microcracks. More specifically, the load carrying ability of a ceramic part is dependent on the relationship between the flaw size distribution and the stress distribution in the part under consideration. If flaws of critical size exist in volume elements that are stressed at or above a critical stress level, fracture will occur. If those same flaws are present in the part, but do not exist in volume elements that are stressed at levels above the critical stress, the part will not fail. Weibull analysis provides a means to statistically evaluate a given design/material/loading combination to determine the likelihood of failure based on the probability that critical flaws will exist in regions of the part where tensile stresses are high.

Most manufacturers of structural ceramics provide Weibull moduli in their advertising literature. The Weibull modulus provides an indication of the consistency of the modulus of rupture, much as a standard deviation provides an indication of repeatability for a metallic material tensile strength. However, one of the shortcomings in ceramic vendor literature is that there is no explanation of how to use the Weibull modulus to evaluate a ceramic part design. The following paragraphs explain how a Weibull modulus is obtained from MOR data, after which the techniques for utilizing the Weibull parameters to evaluate a given part design are outlined.

The first step in conducting a Weibull analysis involves statistically analyzing the distribution of MOR strength values obtained from a large number of tests on the same material to determine the Weibull modulus, m, for the material. The calculated strength values are sorted in ascending order. Each MOR value is assigned a ranking, j. Values for j range from 1 through n, where n is the number of specimens. The probability of failure P_f is then estimated for each specimen. The estimate of P_f can be calculated a number of ways:

$$P_f = \frac{j}{n+1}, \left[P_f = \frac{j-0.5}{n}\right], P_f = \frac{j-0.3}{n+0.4}, \text{ or } P_f = \frac{j-3/8}{n+1/4} \quad [4]$$

The estimate equation in brackets has been demonstrated to be of low bias as long as the number of specimens is greater than 20, and is generally considered to be superior to the others.[2]

There are two types of Weibull analyses that can be performed. The most general type is the 3-parameter Weibull analysis, which takes into account the fact that there is a minimum stress, σ_u, below which the material will never fail. The alternate, 2-parameter analysis assumes that σ_u is equal to zero, implying that failure is possible at any stress. The 2-parameter analysis is generally utilized because the mathematics are less rigorous and the results are more conservative than the 3-parameter analysis.

The general form of the 2-parameter Weibull cumulative distribution function (CDF) is:[3]

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^m\right] \quad [5]$$

This equation can be rewritten as follows:

$$\frac{1}{1-F(x)} = \exp\left(\frac{x}{\theta}\right)^m \quad [6]$$

Taking the natural logarithm of both sides yields:

$$\ln\frac{1}{1-F(x)} = \left(\frac{x}{\theta}\right)^m \quad [7]$$

Taking the natural logarithm of both sides again yields an equation of the form $y = mx + b$:

$$\underbrace{\left[\ln\ln\frac{1}{1-F(x)}\right]}_{y} = \underbrace{m[(\ln x)]}_{mx} - \underbrace{[(m\ln\theta)]}_{b} \quad [8]$$

Substituting P_f for $F(x)$, MOR for x, and σ_0 for θ results in the following equation:

$$\ln \ln \frac{1}{1 - P_f} = m(\ln MOR) - (m \ln \sigma_0) \quad [9]$$

Therefore, a 2-parameter Weibull analysis is usually performed on material data by plotting the following parameters:

$$x = \ln(MOR),\ y = \ln \ln \frac{1}{1 - P_f} \quad [10]$$

This yields a straight-line plot of slope m and y-intercept $m\ ln(\sigma_0)$.

Another quantity related to Weibull statistical analysis is the hazard rate (sometimes called the damage function). The general expression for the hazard rate is as follows:[3]

$$\gamma = hazard\ rate = \frac{f(x)}{1 - F(x)} \quad [11]$$

where $f(x)$ represents the density function and $F(x)$ represents the cumulative distribution function. For Weibull distributions, the hazard rate is represented by the following general equation:

$$\gamma = \frac{m}{\theta^m}(x)^{m-1} \quad [12]$$

The hazard rate is the probability that a given item will fail in the next incremental increase in x (time, stress, etc.) when it has already survived up to x.

For ceramic property applications, x is replaced by σ, and θ is replaced by σ_0, yielding the following equation:

$$\gamma = \frac{m}{\sigma_0^m}(\sigma)^{m-1} \quad [13]$$

In this form of the equation, the hazard rate represents the probability that an item will fail if the stress is raised incrementally above σ, assuming it has already survived up to a stress level of σ. Since all structural ceramic materials display a Weibull modulus greater than one, the hazard rate increases as the stress level increases.

Table 1 contains 100 actual modulus of rupture data points from a commercially available ceramic product. The data have been normalized to alter the average MOR and the Weibull modulus in order to prevent identification of the supplier and material grade. The average MOR after normalization is 630 MPa, with data points ranging from 489 MPa to 721 MPa. Figure 2 is a Weibull plot made from these data. The overall shape of the data plot has not been altered by the data normalization. Examination of the plotted data can reveal several things.

If the Weibull analysis is valid, the points will approximate a straight line. The slope of the line is the estimated Weibull modulus (m_e) for the material. Because of its ease and accuracy compared with graphical techniques, linear regression is generally used to determine the slope. In some cases, the linear regression is run on the data without making a plot. The danger in not examining a plot of the data is that valuable information concerning the data can be overlooked.

For example, the plot can be used to identify points that do not "fit in" with the other points. These points are sometimes called "stray" or "outlier" points. They generally appear as abnormally low MOR values resulting from specimens which contain unusual defects, i.e., defects which are not typical for that material. One of the points shown in Fig. 2 is identified as an outlier point (see arrow). In order for Weibull analysis to be

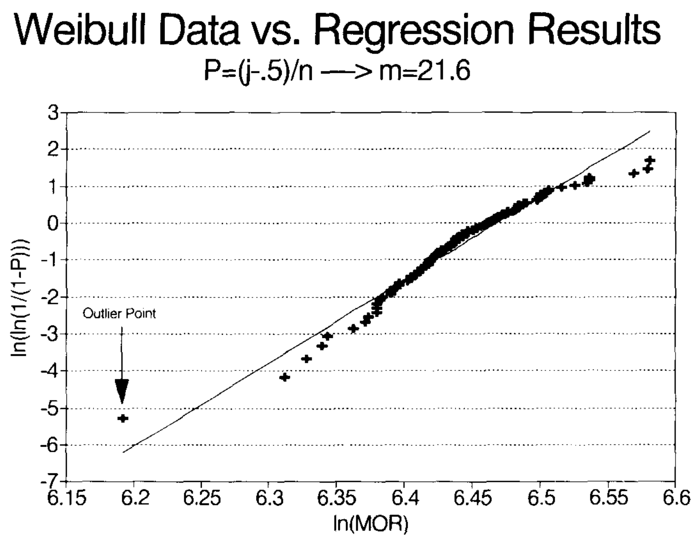

Fig. 2 Weibull plot of the normalized modulus of rupture data.

Table 1 Normalized modulus of rupture data used in example

#	MOR	#	MOR	#	MOR	#	MOR	#	MOR	#	MOR	#	MOR	#	MOR	#	MOR	#	MOR
1	489	11	590	21	605	31	614	41	623	51	629	61	639	71	648	81	655	91	670
2	551	12	591	22	608	32	615	42	623	52	629	62	640	72	649	82	658	92	670
3	560	13	593	23	609	33	616	43	624	53	629	63	640	73	650	83	659	93	676
4	566	14	596	24	609	34	616	44	624	54	631	64	641	74	650	84	664	94	683
5	569	15	596	25	610	35	616	45	625	55	631	65	643	75	653	85	664	95	689
6	580	16	599	26	610	36	618	46	625	56	634	66	644	76	654	86	665	96	690
7	585	17	600	27	611	37	619	47	625	57	634	67	644	77	655	87	666	97	690
8	586	18	600	28	613	38	619	48	625	58	635	68	645	78	655	88	666	98	713
9	590	19	604	29	613	39	620	49	626	59	635	69	646	79	655	89	666	99	720
10	590	20	605	30	614	40	621	50	629	60	638	70	646	80	655	90	669	100	721

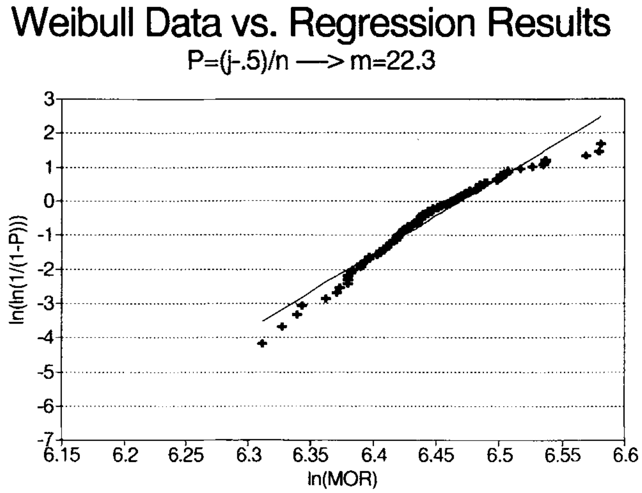

Fig. 3 Weibull plot of the same data from Fig. 2 except the stray point has been removed and the slope has been recalculated.

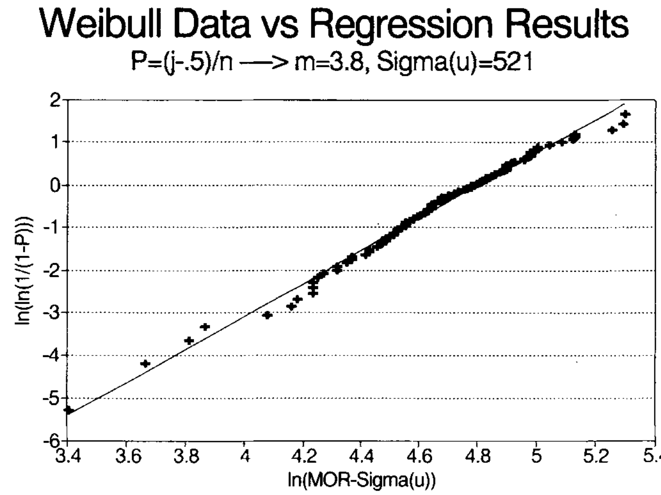

Fig. 4 3-parameter Weibull plot of the same sample data that were plotted in Fig. 3.

valid, the flaw size distribution must be unimodal. Bimodal and multimodal flaw size distributions render Weibull analysis unsuitable. In many cases, fractography of the specimen which produces an outlier point can be used to determine the cause of the abnormally low fracture stress, and non-destructive examination (NDE) techniques, proof testing, and/or process improvements can be implemented to prevent shipment of products containing those types of defects.[1] In this way, the low MOR values can be eliminated from the analysis, which in essence raises the overall Weibull modulus value and improves the validity of the Weibull analysis technique. Figure 3 is the same data plotted after removal of the outlier point from the regression analysis. Note that the calculated Weibull modulus (slope) increased from 21.6 to 22.3.

Another thing that can be learned from examining the plot is whether the 2-parameter Weibull analysis provides an accurate statistical model. Note that the plotted data display a noticeable downward curvature. This indicates that a 3-parameter Weibull analysis will provide a more accurate representation of the data than the 2-parameter analysis.[3] The 3-parameter analysis is performed by making a "best guess" approximation of σ_u, the minimum stress at which failure can occur. That value is subtracted from each MOR value before performing the above calculations. If the resulting plotted line is still curved downward, the estimated σ_u is too small, and a higher value should be tried. If the plotted line is curved upward, the estimated σ_u is too large. The correct value for σ_u results in a straight line, the slope of which is the 3-parameter Weibull modulus. Figure 4 represents the same data as that shown in Fig. 3, with the outlier point deleted, assuming a σ_u of 521 MPa. Note that the data much more closely approximate a straight line. Also note that the 3-parameter Weibull modulus is much lower (3.8 vs. 22.3). This underscores the fact that the 3-parameter Weibull modulus must be considered and utilized in conjunction with its associated σ_u value, and is not directly comparable with the 2-parameter Weibull modulus.

As a standard practice, most of the suppliers of structural ceramics are advertising Weibull moduli which are derived using 2-parameter techniques. Weibull moduli values listed in advertising literature should be identified as such. As stated above, the use of the 2-parameter Weibull analysis will provide conservative results in the evaluation of an engineering design, so its use is acceptable even though it is not as accurate as a 3-parameter analysis in some cases. All of the subsequent discussion will pertain to only 2-parameter Weibull analysis, although the data presented above show that there may be justification for the use of 3-parameter Weibull analysis for some materials.

Design Evaluation Using Weibull Statistics

The Weibull modulus can help the end user sort through the available ceramic materials when trying to identify the best material for a given design. If two materials display the same average MOR, the material with the highest Weibull modulus will provide a more reliable part. On the other hand, if two materials display the same Weibull modulus, the material with the highest average MOR will provide a more reliable part. Clearly, improvement in reliability can be obtained by increasing either the average MOR or the Weibull modulus. Note that a *higher* Weibull modulus implies less data scatter. The opposite is true with respect to standard deviation, where *lower* values mean less data scatter.

As stated above, the average ceramic supplier provides the above information in its advertising literature. However, the actual techniques for using the Weibull modulus in quantitative evaluation of engineering designs are not discussed in the advertising literature, and are not familiar to most engineers. For this reason, they often fall back to philosophies used in the design of metallic parts, involving tensile strengths and safety factors. Some of the ceramic suppliers even publish tensile strength values for their materials, probably with this type of design philosophy in mind. Unfortunately, the tensile strength measurement of a ceramic specimen is a probabilistic value dependent upon the tensile volume of the specimen. The measurement of one or even a few specimens does not provide adequate design data. The publication of a representative tensile strength value is inaccurate at best, and could possibly be construed as misleading.

Weibull analysis allows the calculation of probability of failure in components other than MOR bars. Three requirements must be met for the application of Weibull analysis to be valid:

1. The component must exhibit the same flaw size distribution as the MOR bars. Flaw size distribution is dependent upon a number of manufacturing variables. The manufacturer should be responsible for informing the purchaser if this is not the case.
2. The component must possess the same mechanical properties as the MOR bars. Strength and toughness could be affected by part section size, especially in some of the transformation toughened materials which obtain their properties through heat treatment. The manufacturer should be responsible for informing the purchaser if the mechanical properties will be different in actual components.
3. The stress distribution in the entire component must be known. In all but the simplest components, this requires the use of finite element stress analysis.

Before the Weibull analysis can be performed, several parameters must be obtained from the supplier of the ceramic material:

σ_{mean} Average modulus of rupture (may be called flexural strength, bending strength, etc.). Suggested unit is megapascals (MPa). This is usually available in product literature.

m_e Estimated Weibull modulus (dimensionless). This is usually available in product literature.

n Number of MOR values used to compute Weibull modulus (dimensionless). This information usually must be requested from the vendor.

b Width of MOR test bar (refer to Fig. 1). Suggested unit is meters. This information usually must be requested from the vendor.

h Height (thickness) of MOR test bar (refer to Fig. 1). Suggested unit is meters. This information usually must be requested from the vendor.

L_1 Distance between support points in MOR load fixture (major span, refer to Fig. 1). Suggested unit is meters. This information usually must be requested from the vendor.

L_2 Distance between load points in MOR load fixture (minor span, refer to Fig. 1). Suggested unit is meters. This information usually must be requested from the vendor. (This value is 0 if the MOR is determined in 3-point bending.)

Care must be taken to ensure that units for all of these parameters agree. It is recommended that the SI (meters-kilograms-seconds) system of measures be utilized for all calculations, as shown above. Use of inconsistent units results in erroneous solutions.

Because of differences among various suppliers in the number of specimens used for computation of the Weibull modulus and the dimensions used in MOR testing, the estimated Weibull modulus (m_e) and average rupture strength (σ_{mean}) values cannot be used directly in a design evaluation or even for accurate comparison of materials. A number of steps must be taken to normalize those values to ensure that data from all suppliers are utilized in an equivalent manner.

The uncertainty associated with the determination of the Weibull modulus (m) is a function of the number of MOR values utilized. In order to account for this uncertainty, the Weibull modulus used in the subsequent steps should be adjusted downward to account for statistical spread in accordance with the number of samples the vendor used in the determination of the reported Weibull modulus. The 90%, 95%, and 99% confidence values for m (actually, m_{min}) can be estimated by the following formula:

$$m = \frac{m_e}{f} \qquad [14]$$

where m_e is the supplier's reported Weibull modulus and f is the factor given in Table 2.[2]

The factor f should be chosen based on the number of specimens used by the manufacturer to determine the estimated Weibull modulus and the degree of confidence that is considered necessary for the application being evaluated. The resulting m value should be used in all equations that follow.

Because MOR tests are run using a variety of different fixture dimensions, the volume under stress is not constant from one manufacturer to another. This affects the value of σ_{mean}, since the probability of failure at a given load is small if the stressed volume is small. The normalized strength value which must be calculated is the characteristic stress per unit volume (σ_0). The following steps are utilized to obtain σ_0.

The characteristic strength of MOR bars (MOR_0, the stress level in the outer fiber of the bar at or below which 63% of the bars will fail) is calculated using the following equation:

$$MOR_0 = \sigma_{mean} \times A, \text{where } A = \left[\frac{1}{\Gamma\left[\frac{1}{m} + 1\right]}\right] \qquad [15]$$

Values for A vs. m are given in Table 3.

Table 2 $f = m_e/m$

n	Confidence interval 90%	95%	99%
10	1.48	1.67	2.18
20	1.30	1.40	1.62
50	1.18	1.24	1.35
100	1.12	1.16	1.23
200	1.08	1.11	1.15
500	1.04	1.07	1.10
1000	1.03	1.04	1.07

Table 3 Values of A as a function of Weibull modulus, m

m	A	m	A	m	A	m	A
1	1.00000	16	1.03351	31	1.01794	46	1.01224
2	1.12838	17	1.03168	32	1.01740	47	1.01198
3	1.11985	18	1.03004	33	1.01689	48	1.01174
4	1.10327	19	1.02856	34	1.01640	49	1.01151
5	1.08912	20	1.02722	35	1.01596	50	1.01129
6	1.07791	21	1.02600	36	1.01553	51	1.01106
7	1.06902	22	1.02487	37	1.01513	52	1.01086
8	1.06186	23	1.02386	38	1.01473	53	1.01065
9	1.05600	24	1.02291	39	1.01437	54	1.01047
10	1.05114	25	1.02204	40	1.01402	55	1.01027
11	1.04703	26	1.02123	41	1.01368	56	1.01010
12	1.04353	27	1.02048	42	1.01337	57	1.00993
13	1.04051	28	1.01977	43	1.01307	58	1.00975
14	1.03787	29	1.01912	44	1.01278	59	1.00959
15	1.03556	30	1.01851	45	1.01250	60	1.00944

Note that since MOR_0 is the stress at which 63% of the bars will fail, MOR_0 will be greater than σ_{mean}, the stress at which approximately 50% of the bars will fail.

The characteristic stress per unit volume (σ_0, a Weibull parameter) is then calculated:[4]

$$\sigma_0 = MOR_0\left(\frac{bh}{2}\right)^{1/m}\left[\frac{L_1 + mL_2}{(m+1)^2}\right]^{1/m} \qquad [16]$$

The values σ_0 and m can now be used to compare materials from different suppliers, because they are theoretically independent of the testing geometry and the number of specimens tested. Although it is currently not the case, these values should be made available in the product literature from ceramic material suppliers.

The probability of failure for a part is yielded by the following 3-parameter Weibull equation:

$$P_f = 1 - \exp\left[-\int_v \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m dV\right] \qquad [17]$$

where

σ = the stress in differential volume dV

σ_u = the stress below which fracture will not occur

σ_0 = the 3-parameter characteristic strength/unit volume

m = the 3-parameter Weibull modulus

Since 3-parameter data is not generally available in most practical applications, the 2-parameter form of the equation is usually used:

$$P_f = 1 - \exp\left[-\int_v \left(\frac{\sigma}{\sigma_0}\right)^m dV\right] \qquad [18]$$

where

σ = the stress in differential volume dV

σ_0 = the 2-parameter characteristic strength/unit volume

m = the 2-parameter Weibull modulus

If the entire volume of the part is under uniaxial tensile stress, the equation can be simplified as follows:

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m V\right] \qquad [19]$$

where V = volume of part.

Use of Eq 19 assumes that the stress is equivalent throughout the entire volume. This equation can be used for parts which are not under uniform uniaxial tensile stress by assuming that the entire volume is at the peak stress existing within the part. However, this is a very conservative approach, and results in predicted failure rates that are usually much higher than the actual probability of failure.

Equation 19 can be rearranged to show the effect of volume on the reliability of ceramic components:

$$\sigma = \sigma_0\left[\frac{-\ln(1 - P_f)}{V}\right]^{1/m} \qquad [20]$$

Figure 5 is a plot of the above equation showing the maximum design stress as a function of stressed volume for various probabilities of failure. Note that as the stressed volume increases, the probability of failure increases for a given design stress.

Since few parts are stressed in simple uniaxial tension, Eq 19 can seldom be utilized. In addition, most actual components have stress distributions which do not lend themselves to simple mathematical representation and/or integration. In these cases, finite element stress analysis must be utilized to determine maximum stresses in each of a number of volume elements in the part. Pseudo-integration techniques can then be used to perform the Weibull analysis. Equations 21 and 22 provide the mathematical basis for this technique:

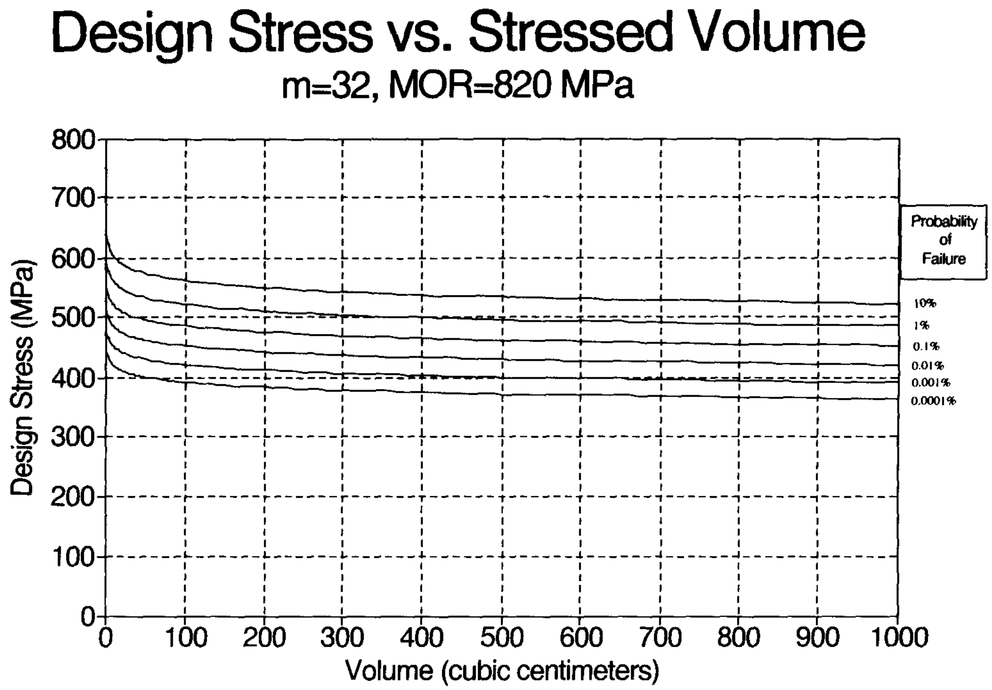

Fig. 5 Allowable design stress vs. stressed volume for various probabilities of failure.

$$P_f = 1 - \exp\left[-\sum_{i=1}^{n}\left(\frac{\sigma_i - \sigma_u}{\sigma_0}\right)^m \Delta V_i\right], \sigma_i \geq \sigma_u \quad [21]$$

where
n = the total number of elements
V_i = the volume of volume element i
σ_i = the stress in volume element i
σ_u = the stress below which fracture will not occur
σ_0 = the 3-parameter characteristic strength/unit volume
m = the 3-parameter Weibull modulus

Assuming $\sigma_u = 0$, the 2-parameter form of this equation becomes:

$$P_f = 1 - \exp\left[-\sum_{i=1}^{n}\left(\frac{\sigma_i}{\sigma_0}\right)^m \Delta V_i\right], \sigma_i \geq 0 \quad [22]$$

where
n = the total number of elements
V_i = the volume of volume element i
σ_i = the stress in volume element i
σ_0 = the 2-parameter characteristic strength/unit volume
m = the 2-parameter Weibull modulus

It should be noted that elements under compressive stress must be excluded from the computation, since only elements under tensile stress contribute toward failure of the component. The output of this analysis is the probability that the component will fail under the loading conditions defined in the finite element analysis model. In order for this technique to provide accurate results, several conditions must be satisfied:

- An adequate number of elements must be used in the finite element model, particularly in locations which are highly stressed.
- The use of finite element output values which are averages of stresses throughout the element, such as principal stress at the centroid, should not be utilized for σ_i. Instead, integration point stresses should be utilized. Otherwise, large stress gradients may exist across individual elements, and the value input as σ_i in the Weibull analysis may not provide an accurate estimate of each element's contribution toward probability of failure.[5]
- The loading conditions and geometry utilized in the model must accurately describe the component and the loading it will actually experience. Since the equation for estimating probability of failure contains several power functions, errors in stress levels and/or Weibull modulus estimates can produce very large errors in the estimated probability of failure. This effect becomes more pronounced as the Weibull modulus increases. Since many of the newer ceramic materials have Weibull moduli greater than 20, this effect should not be overlooked.
- The material properties and Weibull modulus for the component must agree with the values used in the calculations.

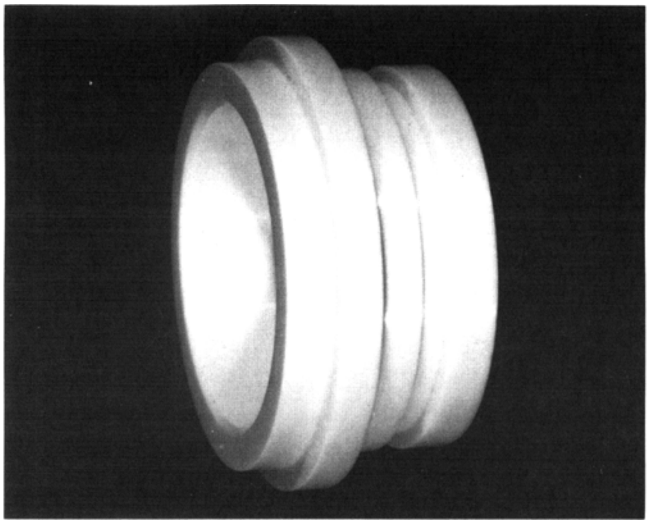

Fig. 6 Photograph of valve seat ring used in finite element/Weibull analysis example.

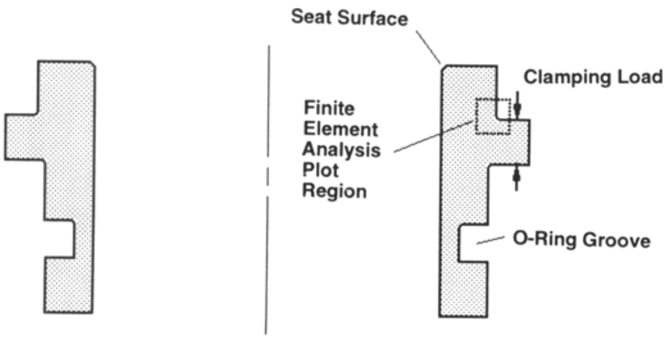

Fig. 7 Sketch of seat ring cross-section showing seat surface, O-ring groove, and cage clamping location. Finite element analysis plot location is also identified.

The designer must determine whether the calculated probability of failure is acceptable for the component in question based upon the consequences surrounding failure of the component. If it is determined that the design/loading/material combination results in a probability of failure which is unacceptably high, there are several options available to reduce the probability of failure:

1. Utilize a material with a higher average strength, all other factors being equal.
2. Utilize a material with a higher Weibull modulus, all other factors being equal.
3. Redesign the component to reduce peak stresses.

Example Finite Element / Weibull Analysis

Figure 6 is a photograph of a ceramic control valve seat ring. Figure 7 is a sketch showing the cross-section of the seat ring.

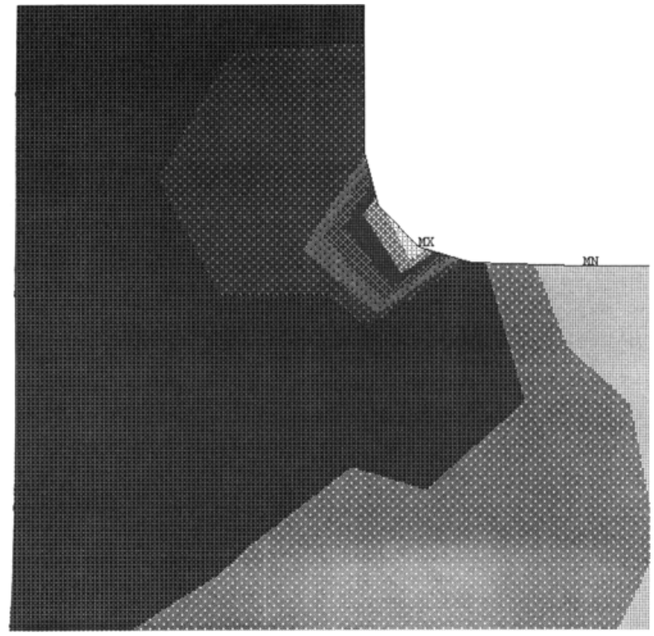

Fig. 8 Finite element stress plot of seat ring location outlined in Fig. 7. Note the maximum stress location labeled "MX."

The flange on the O.D. of the seat ring is clamped between a cage and the valve body. An O-ring in the groove on the O.D. seals the leak path between the seat ring and the valve body. During operation, a valve plug is brought into contact with the bevel on the upper I.D. of the seat ring to provide shutoff.

A finite element analysis was performed on the seat ring design to determine the magnitude and distribution of stresses in the seat ring under different conditions of operation. Figure 8 is a finite element plot of the stresses present in a portion of the seat ring due to cage clamping load and internal pressure, but without any plug load on the seating surface. The maximum stress is located at the root of the clamping flange, as indicated by the "MX" on the plot.

Two ceramic materials that were candidates for this part possessed the properties summarized in Table 4.

Equation 14 and Table 2 were used to determine m as a function of m_e. A 95% confidence in the value of m was desired, which resulted in the values calculated below:

$$\text{Material 1: } m = \frac{m_e}{f} = \frac{32}{1.16} = 27.6 \approx 28$$

$$\text{Material 2: } m = \frac{m_e}{f} = \frac{20}{1.04} = 19.2 \approx 19$$

MOR_0 for each material was then calculated according to Eq 15 and Table 3:

$$\text{Material 1: } MOR_0 = \sigma_{mean} \times A = 820 \times 1.01977 = 836 \text{ MPa}$$

$$\text{Material 2: } MOR_0 = \sigma_{mean} \times A = 620 \times 1.02856 = 638 \text{ MPa}$$

The characteristic strength per unit volume, σ_0, for each material was determined using Eq 16:

Material 1:

$$\sigma_0 = MOR_0\left(\frac{bh}{2}\right)^{1/m}\left[\frac{L_1 + mL_2}{(m+1)^2}\right]^{1/m}$$

$$= 836\left(\frac{0.00635 \times 0.003175}{2}\right)^{1/28} \times \left[\frac{0.01905 + (28 \times 0.009525)}{(28+1)^2}\right]^{1/28} = 417$$

Material 2:

$$\sigma_0 = MOR_0\left(\frac{bh}{2}\right)^{1/m}\left[\frac{L_1 + mL_2}{(m+1)^2}\right]^{1/m}$$

$$= 637\left(\frac{0.00635 \times 0.003175}{2}\right)^{1/19} \times \left[\frac{0.039878 + (19 \times 0.020066)}{(19+1)^2}\right]^{1/19} = 243$$

A computer program was written to calculate the probability of failure according to Eq 22, utilizing the values calculated for m and σ_0 in conjunction with an array of stress and volume values from the finite element analysis. The resulting probabilities of failure were 0.00036 for Material 1 and 0.39 for Material 2. It was concluded that the resulting failure rate for Material 1 was acceptable for this loading configuration. Other loading configurations were also examined, with similar results. Therefore, Material 1 was used for the part without design modification. If Material 2 were to be utilized, significant modifications in design and/or reduction in load values would be required to attain an acceptable failure rate.

The same program and finite element analysis data set were used to produce a matrix of values demonstrating the effects of average MOR and Weibull modulus on the probability of failure. The results are listed in Table 5 and plotted in Fig. 9.

The data and plot demonstrate the concept that increasing either the Weibull modulus or the average strength will decrease the probability of failure. However, the plot also demonstrates a weakness in the calculation of probability of failure. Note that for the strength/Weibull combinations which result in probabilities of failure greater than approximately 0.6, the calculated probability curves differ in shape from the curves for lower probability combinations. This is because the finite ele-

Table 4 Candidate materials

	Material 1	Material 2
Reported properties		
Average MOR, σ_{mean}, MPa	620	820
Estimated Weibull modulus, m_e	20	32
Testing parameters		
Minor span (L_1), meters	0.009525	0.02007
Major span (L_2), meters	0.01905	0.03988
Bar width (b), meters	0.00635	0.00635
Bar height (h), meters	0.003175	0.003175
Number of specimens, n	100	1000

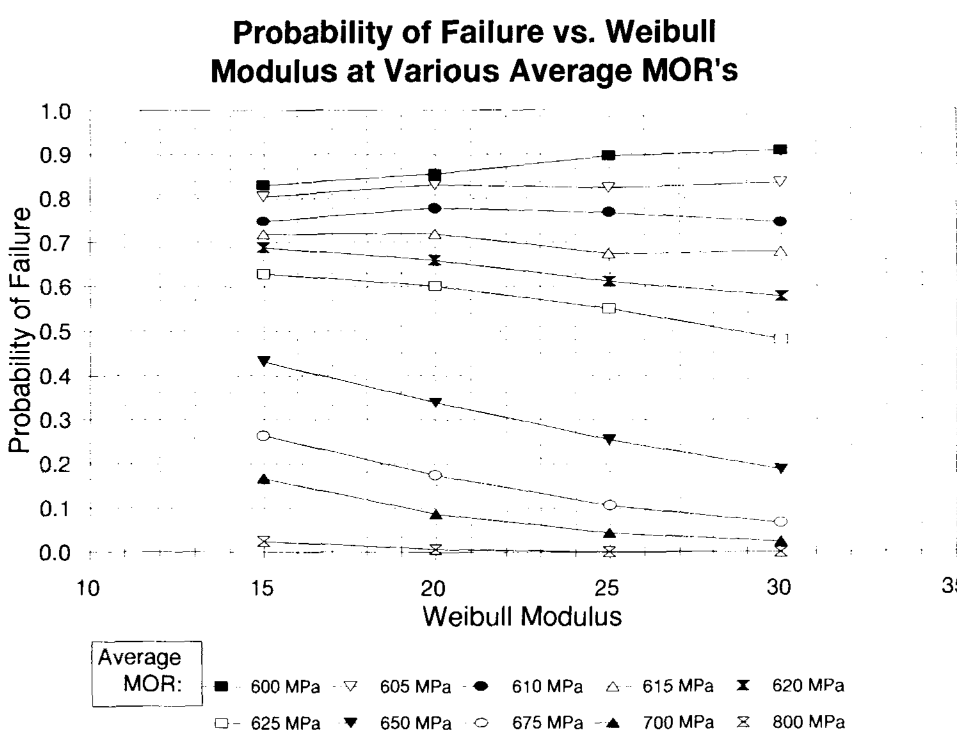

Fig. 9 Plot of probability of failure vs. a number of hypothetical σ_{mean}/m combinations.

Table 5 Probability of failure for seat ring under particular loading conditions for a variety of hypothetical MOR_{mean}/m combinations

Average MOR, MOR_{mean}, MPa	Probability of failure Weibull modulus, m			
	15	20	25	30
600	0.830	0.855	0.896	0.908
605	0.804	0.831	0.824	0.835
610	0.748	0.778	0.768	0.745
615	0.719	0.720	0.675	0.679
620	0.689	0.660	0.612	0.578
625	0.629	0.601	0.550	0.481
650	0.432	0.339	0.255	0.187
675	0.264	0.175	0.106	0.0660
700	0.167	0.0878	0.0435	0.0232
800	0.0249	0.00651	0.00159	0.000398

ment analysis data set contains a significant number of volume elements which have σ_i values that are greater than σ_0 for the lower strength materials. When σ_i is greater than σ_0, the term

$$\left(\frac{\sigma}{\sigma_0}\right)^m$$

in Eq 22 becomes very large. If too many of the volume elements produce these large terms, the overall expression begins to "misbehave." Therefore, it is concluded that the equations for predicting probability of failure are only accurate when the values for σ_i are predominantly less than σ_0. This phenomenon is of little practical significance, since it will only affect analyses where the resulting probability of failure is too high to be considered acceptable. For resulting probabilities which are in the acceptable or near-acceptable range, the equations produce results that are consistent with theory.

Some General Design Guidelines

There are a number of guidelines that should be considered when designing ceramic components. Those guidelines can be divided into two categories: those based on properties of the ceramic, and those based on manufacturing techniques and associated costs.

Property-based guidelines:

- Ceramics behave somewhat like very brittle metals, although usually exhibiting even less toughness than the most brittle metals. Therefore, design precautions followed for brittle metal parts are also appropriate for ceramic materials.
- Ceramics have very high compressive strength. They are not as reliable in tension. Parts should be designed to avoid tensile stresses if possible.
- Generous radii or blending should be specified where section changes occur. Sharp section changes result in stress concentrations, greatly increasing probability of failure.
- Point and/or line contacts should be avoided because of small regions of high tensile stress created in adjacent material.
- Stresses induced at joints with metallic parts must be critically evaluated. Intense local stresses can be created at these joints, especially when differential thermal expansion is involved.
- Impact loading of ceramic components is not recommended.
- If tensile stresses are unavoidable, they should be kept as low as possible.

Manufacturing-based guidelines:

- Specify tight tolerances only where necessary. Tolerances tighter than 1% of the overall dimension (a rule of thumb that might vary depending on the type of ceramic and manufacturing techniques) require diamond grinding after the sintering operation, which substantially increases costs.
- Keep shapes as simple as possible.
- Grinding of simple shapes on outside surfaces, O.D.'s, ends, etc., allows use of large, high-speed grinding wheels while still providing for adequate coolant access. This facilitates rapid removal of material, resulting in low added cost.
- Grinding of inside features or complex shapes on outside surfaces, where access is limited to small grinding wheels, reduces the speed at which material can be removed. Inside grinding also provides restricted coolant access, which can slow grinding even further. Thus, grinding of inside surfaces and/or complex shapes on outside surfaces results in high added cost.

Summary

Ceramic materials offer unique properties which make them attractive for a wide variety of applications in many industries, but they have been utilized in only a small percentage of those potential applications. Their lack of use can be attributed to some of their inherent detrimental properties such as brittleness and lack of apparent tensile strength, as well as to a general unfamiliarity with the design practices which permit quantitative analysis of the reliability of ceramic components.

Weibull analysis allows quantitative evaluation of probability of failure in ceramic components. Calculation of the probability of failure for simple parts and stress distributions is fairly straightforward, but since most actual components have stress distributions which do not lend themselves to simple mathematical representation and/or integration, finite element stress analysis and pseudo-integration techniques must generally be utilized to perform Weibull analysis. Whereas in the recent past the high cost and lack of access to finite element analysis systems would have deemed Weibull analysis prohibitive for many companies, the current availability and low cost of computers and software for engineering analysis makes Weibull analysis readily available to the engineering community for the design of reliable ceramic components.

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